

# **Skills from previous math classes that you need to self-review for Math 2A**

## **From Algebra:**

Negative and fractional exponents  
Rational expressions  
    Add / subtract  
    Polynomial long division

## **From Precalculus:**

Finding zeros & factoring polynomials  
Variation / proportionality  
Recurrence relations

## **From Calculus:**

Limits involving infinity  
Continuity & differentiability (single & multivariable functions)  
Differentiation (basic, product / quotient / chain rule)  
Interpretation of derivatives (and their relationship to increasing/decreasing behavior of functions)  
Integration (basic, substitution, by parts, partial fraction decomposition, improper, multivariable)  
Power series

**You must be able to solve these**  
**using neither your calculator nor a table of integrals**  
**All answers must be completely simplified**

[1] Over what interval(s) are the following functions continuous ? Over what interval(s) are they differentiable ?

- |                                       |                      |                              |                              |
|---------------------------------------|----------------------|------------------------------|------------------------------|
| [a] $f(x) = x^3$                      | [b] $f(x) = x^{-2}$  | [c] $f(x) = x^{\frac{1}{3}}$ | [d] $f(x) = x^{\frac{3}{2}}$ |
| [e] $f(x) = e^{3x}$                   | [f] $f(x) = 2^{-3x}$ | [g] $f(x) = \ln x$           |                              |
| [h] $f(x) = \sin x$                   | [i] $f(x) = \tan x$  | [j] $f(x) = \csc x$          |                              |
| [k] $f(x) = \frac{x+1}{(x-2)(x^2+9)}$ |                      |                              |                              |

- [2] [a] If  $y = x^2 e^{-3x}$ , find  $\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} - 2y$ .
- [b] If  $y = x^{\frac{4}{3}}$ , find  $3x^2 y'' - 4xy' + 4y$ . For what values of  $x$  is your answer correct ?
- [c] If  $y = Ae^{2x} \sin 3x + Be^{2x} \cos 3x$ , find  $\frac{d^3 y}{dx^3} - 4 \frac{dy}{dx} - 9y$ .

- [3] Factor each polynomial below and also
- |                              |  |
|------------------------------|--|
| [a] $3x^3 - 10x^2 + 7x + 10$ | [i] into a product of linear and/or irreducible quadratic factors, |
| [b] $6x^5 - 5x^4 - 24x + 20$ | [ii] into a product of linear factors                              |

[4] Find all zeros (real and complex) of the polynomial  $x^5(2x^2 - 3x - 1)^2(x^2 - 2x + 3)^3$ , and find the multiplicity of each zero.

[5] Evaluate the following integrals.

- |   |                                  |                                    |  |
|---|----------------------------------|------------------------------------|--|
| [a] $\int e^{-2x} \sin 3x \, dx$            | [b] $\int x^3 \cos 2x \, dx$     | [c] $\int \frac{1}{1+4x} \, dx$    | [d] $\int \frac{1}{4+x^2} \, dx$           |
| [e] $\int \frac{1}{4-x^2} \, dx$            | [f] $\int \frac{x}{4-x^2} \, dx$ | [g] $\int \frac{x^2}{4-x^2} \, dx$ | [h] $\int \frac{3x^4 - 5x^3}{4-x^2} \, dx$ |
| [i] $\int \frac{100}{(4+x)^2(4+x^2)} \, dx$ | [j] $\int \frac{x^3}{y} \, dy$   | [k] $\int \frac{x^3}{y} \, dx$     |  |

- [6] [a] If  $-\ln y = 2 \ln x + C$ , and  $y = 4$  when  $x = 3$ , write  $y$  as a function of  $x$ .
- [b] If  $y(x) = Ae^{2x} \sin 3x + Be^{2x} \cos 3x$ , and  $y(\frac{\pi}{2}) = -4$  and  $y'(\frac{\pi}{2}) = 1$ , find the values of  $A$  and  $B$ .
- [c] If  $\frac{dy}{dx} = \frac{1}{\sqrt[3]{x}}$  and  $y(8) = -1$ , write  $y$  as a function of  $x$ .

[7] If  $f(x, y) = \frac{3y-2x}{5x+4y}$ , find  $f_x$  and  $f_y$ . Find all  $(x, y)$  where  $f_y$  is continuous.

[8] If  $\frac{\partial f}{\partial x} = \frac{3y-2x}{5x+4y}$ , find  $\frac{\partial^2 f}{\partial x \partial y}$  and  $\frac{\partial^2 f}{\partial y \partial x}$ .

[9] If  $f'(x) = \frac{e^x}{x}$ , and  $f(2) = -1$ , find the slope-point equation of the tangent line to  $y = f(x)$  at  $x = 2$ .

- [10] Write equations corresponding to the following situations. **(NOTE: Do NOT solve the resulting equations.)**
- [a] The net wealth of a person is proportional to the cube of their age.
  - [b] The reaction time of a subject varies inversely with the square root of their weight.
  - [c] The rate of change of a population of rabbits varies directly with the square of the population.
  - [d] The rate at which you acquire knowledge in a subject varies directly with the difference between your current knowledge and the total knowledge possible (which is a fixed constant).

- [11] If  $u = \ln y$ , use the chain rule to write  $\frac{dy}{dx}$  in terms of  $x, u$  and  $\frac{du}{dx}$ .

**(NOTE: Your final answer must NOT involve  $y$ .)**

- [12] If  $\frac{dy}{dx} = \frac{4-x^2}{(x+1)^2}$ , find all intervals over which  $y$  is increasing.

**(NOTE: Do NOT solve for  $y$ .)**

- [13] Find  $\int_0^{\infty} x^2 e^{-3x} dx$ .

- [14] If  $a_{n+2} = na_{n+1} - 3a_n$  for  $n \in \mathbb{Z}^{non-neg}$ , and  $a_0 = -1$  and  $a_1 = 2$ , find  $a_4$ .

- [15] Find the horizontal asymptote(s) of  $f(x) = \frac{7}{3+2e^{-5x}}$ .

- [16] [a] Write the first five non-zero terms of the power series  $\sum_{n=1}^{\infty} \frac{7 \cdot 4 \cdot 1 \cdots (10-3n)}{(n-1)!} x^{2n}$ .

- [b] Write the power series  $\frac{2}{1 \cdot 2}x - \frac{4}{1 \cdot 2 \cdot 3}x^3 + \frac{8}{1 \cdot 2 \cdot 3 \cdot 4}x^5 - \frac{16}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}x^7 + \frac{32}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}x^9 - \cdots$  in sigma notation using 0 as the lower limit of summation.

### **HINT:**

The questions above involve material from the following classes.

Precalculus I:	Math 41	[3], [4], [6], [10]
Precalculus III:	Math 43	[14]
Calculus I:	Math 1A	[1], [2], [6], [9], [11], [12], [15]
Calculus II:	Math 1B	[5], [6], [9], [13]
Calculus III:	Math 1C	[16]
Calculus IV:	Math 1D	[5], [7], [8]

### **NOTE:**

There is no solution key for this prerequisite package since it only involves material that you have learned before. You are encouraged to work together with your classmates, and to consult your old textbooks and notes. Feel free to ask me to look over your solutions, or to direct you to relevant sections in your old textbooks. However, I will not give solutions to any questions.